III. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Suppose $ABCD$ the required trapezoid, AB being the shorter (upper) base, $BD=x$, one diagonal, $AC=BF=y$, BG=altitude$=h$. Produce DC to F so that $DF=DG+GF=DC+CF=a$, $x+y=c$, θ=given angle. $\sqrt{(x^2-h^2)}+\sqrt{(y^2-h^2)}=a$. $4x^2y^2=a^4+(x^2+y^2)^2+4a^2h^2-2a^2(x^2+y^2)$. But $x^2+y^2=c^2-2xy$. $\therefore 4(c^2-a^2)xy=4a^2h^2+(c^2-a^2)^2$. Now $xy\sin\theta=ah$.

$$\therefore 4a^2h^2-4ah\left(\frac{c^2-a^2}{\sin\theta}\right)=-(c^2-a^2)^2.$$

$$\therefore h=\frac{1}{2a}(c^2-a^2)\tan\tfrac{1}{2}\theta \text{ or } h=\tfrac{1}{2}(c^2-a^2)\cot\tfrac{1}{2}\theta.$$

$$\therefore xy=\frac{c^2-a^2}{4\cos^2\frac{1}{2}a} \text{ or } \frac{c^2-a^2}{\sin^2\frac{1}{2}\theta} \text{ and } x+y=c.$$

$$\therefore x=\tfrac{1}{2}c\pm\frac{\sqrt{(a^2-c^2\cos^2\frac{1}{2}\theta)}}{2\sin\frac{1}{2}\theta} \text{ or } \tfrac{1}{2}c\pm\frac{\sqrt{(a^2-c^2\sin^2\frac{1}{2}\theta)}}{2\cos\frac{1}{2}\theta};$$

$$y=\tfrac{1}{2}c\mp\frac{\sqrt{(a^2-c^2\cos^2\frac{1}{2}\theta)}}{2\sin\frac{1}{2}\theta} \text{ or } \tfrac{1}{2}c\mp\frac{\sqrt{(a^2-c^2\sin^2\frac{1}{2}\theta)}}{2\cos\frac{1}{2}\theta}.$$

Hence lay off $DF=a$, and draw AE parallel to DF at a distance h from it. With D as center and $DB=x$ or y draw DB; then $DF=y$ or x. Draw any line parallel to BF as AC, $A'C'$, $A''C''$, and join B, D to C, C', C'', A, A', A'', respectively. Then any one of the many trapezoids thus formed fulfill the required conditions, as is evident by drawing a figure.

IV. Solution by J. J. KEYES, Nashville, Tenn.

Construct the triangle EBF having BE=sum of diagonals, BF=sum of bases, and angle $E=\frac{1}{2}$ given angle. At F construct the angle EFD=angle E, FD meeting BE in D. Through D draw DM parallel to FB. Take any point C on BF, draw CA parallel to FD meeting DM in A. Join AB, DC. $ABCD$ is the required trapezoid. In proof, $AO=CF$. $\therefore AD+BC=BC+BD$=sum of bases. $AC=DF=DE$. $\therefore AC+BD=BD+DE$=sum of diagonals. The angle between BD and AC=angle $BDF=2$ angle E=given angle.

Also solved by Elmer Schuyler.

CALCULUS.

Problem number 181 was also solved by S. A. Corey and L. C. Walker.

183. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

Evaluate $\int_0^\infty \frac{\sin 2nx dx}{(a^2+x^2)\sin x}$.

Solution by G. B. M. ZERR, A. M., Ph. D.

Let $u=\int_0^\infty \frac{\sin 2nx\, dx}{(a^2+x^2)\sin x}$, then

$$\frac{du}{dn}=\int_0^\infty \frac{2x\cos nx dx}{(a^2-x^2)\sin x},\quad \frac{d^2u}{dn^2}=-\int_0^\infty \frac{4x^2\sin 2nx dx}{(a^2+x^2)\sin x}=4a^2u-4\int_0^\infty \frac{\sin 2nx dx}{\sin x}.$$

$$\text{Let } v=8\int_0^\infty \frac{\sin nx\cos nx\, dx}{\sin x}=8\int_0^\infty \frac{e^{nx\sqrt{-1}}-e^{-nx\sqrt{-1}}}{e^{x\sqrt{-1}}-e^{-x\sqrt{-1}}}\cos nx dx.$$

Let $x\sqrt{(-1)}=\pi y$, $dx=-\pi\sqrt{(-1)}dy$;

$$\therefore v=-8\pi\sqrt{(-1)}\int_0^\infty \frac{e^{\pi ny}-e^{-\pi ny}}{e^{\pi y}-e^{-\pi y}}\cos(\pi ny\sqrt{-1})dy$$

$$=-8\pi\sqrt{(-1)}\left(\frac{\sin\pi n}{e^{\pi n\sqrt{-1}}+2\cos\pi n+e^{-\pi n\sqrt{-1}}}\right)=-2\pi\sqrt{(-1)}\tan\pi n.$$

$\therefore v=0$, since $n=$any integer. $\therefore \frac{d^2u}{dn^2}-4a^2u=0.$

$u=Ae^{2an}+Be^{-2an}$. When $n=0$, $u=0$, and $B=-A$. $\therefore u=A(e^{2an}-e^{-2an})$.

When $n=1$, $u=\frac{\pi e^{-a}}{a}$ and $A=\frac{\pi e^{-a}}{a(e^{2a}-e^{-2a})}$.

$$\therefore u=\frac{\pi e^{-a}}{a}\cdot\frac{e^{2an}-e^{-2an}}{e^{2a}-e^{-2a}}=\frac{\pi e^{-a}}{a}\cdot\frac{\sinh 2an}{\sinh 2a}.$$

184. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

If $u=f(x,y)$; $\xi=e^xy$; $y\eta=e^x$; show that

$$\frac{d^2u}{dx^2}-y^2\frac{d^2u}{dy^2}-y\frac{du}{dy}=4\xi\eta\frac{d^2u}{d\xi.d\eta}.$$

Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

We have $x=\frac{1}{2}\log\xi+\frac{1}{2}\log\eta$, $y=\sqrt{(\xi/\eta)}$, and therefore

$$\frac{\partial}{\partial\xi}=\frac{\partial x}{\partial\xi}\cdot\frac{\partial}{\partial x}+\frac{\partial y}{d\xi}\cdot\frac{\partial}{\partial y}=\tfrac{1}{2}\left(\frac{1}{e^x y}\frac{\partial}{\partial x}+\frac{1}{e^x}\frac{\partial}{\partial y}\right),\text{ whence } 2\xi\frac{\partial}{\partial\xi}=\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}.$$

In a similar way we get

$$2\eta\frac{\partial}{\partial\eta}=\frac{\partial}{\partial x}=y\frac{\partial}{\partial y}.$$